J. Plasma Physics (2010), vol. 76, part 1, pp. 49–56.
doi:10.1017/S0022377809008101

Resonant Weibel instability in counterstreaming plasmas with temperature anisotropies

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(Received 24 February 2009 and accepted 2 April 2009, first published online 19 May 2009)

Abstract. The Weibel instability, driven by a plasma temperature anisotropy, is non-resonant with plasma particles: it is purely growing in time, and does not oscillate. The effect of a counterstreaming plasma is examined. In a counterstreaming plasma with an excess of transverse temperature, the Weibel instability arises along the streaming direction. Here it is proved that for large wave-numbers the instability becomes resonant with a finite real (oscillation) frequency, $\omega_{\rm r} \neq 0$. When the plasma flows faster, with a bulk velocity larger than the parallel thermal velocity, the instability becomes dominantly resonant. This new feature of the Weibel instability can be relevant for astrophysical sources of non-thermal emissions and the stability of counterflowing plasma experiments.

1. Introduction

Electromagnetic (EM) instabilities of the Weibel type [1, 2] release the free energy stored in a velocity anisotropy of plasma particles, whether it is a temperature anisotropy (Weibel instability) [1] or a counterstream (filamentation or Weibel-streaming instability) [2]. Such instabilities have been widely investigated [3–8], being applied in scenarios for the generation of magnetic fields in non-thermal astrophysical sources [9–14] or for the stabilization of the beam–plasma systems in the laboratory [5, 6, 15].

The first systematic analysis of the Weibel instability in a non-streaming plasma was provided in Ref. [3], where there was considered a field-free plasma with a bi-Maxwellian distribution and an arbitrary orientation of the wave-vector. For oblique propagation with respect to the principal temperature axis, the instability is a transverse–longitudinal coupled mode. On the other hand, the first numerical experiments [4] showed that the amplitude saturation for the Weibel instability is a result of magnetic trapping, which occurs once the magnetic bounce frequency reaches a value comparable to the linear growth rate prior to saturation. (The saturation can also be conditioned by an electron gyroradius in the generated magnetic field of the order of the electron collisionless skin depth [5,9].) Moreover, an electrostatic field component arises due to the gradient of the magnetic pressure in the filamentation instability [16], and recent simulations [8,16] led to the conclusion that both fields, magnetic and electric, can thermalize the electrons.

The temperature anisotropy instability originally described by Weibel [1] is non-resonant with plasma particles: it grows exponentially in time, $\mathrm{Im}(\omega) \equiv \omega_{\mathrm{i}} > 0$, and does not propagate, $\mathrm{Re}(\omega) \equiv \omega_{\mathrm{r}} = 0$. Recent studies of counterstreaming plasmas with intrinsic temperature anisotropy [14, 15, 17–20] presented new properties of the Weibel-like unstable modes, different from those for a non-streaming plasma [1]. In Ref. [14] a field-free counterstreaming plasma was considered, and it was shown that the Weibel instability, generated in the streaming direction by an excess of transverse temperature, can be faster than the electrostatic two-stream instability and thus it develops as a primary mechanism of relaxation for the plasma temperature anisotropy. Moreover, when the streaming velocity is sufficiently large, the growth rates exhibit slope breaks for wave-numbers larger than a threshold value. Here, we further investigate this parallel Weibel-like mode that arises only in a counterstreaming plasma with an excess of transverse kinetic energy, and we show that those slope breaks of the growth rates can be attributed to a resonant regime, $\omega_{\mathrm{r}} \neq 0$, of the Weibel instability.

2. Basic kinetic theory

2.1. Plasma with temperature anisotropy

The linearized Vlasov–Maxwell set of equations is commonly used to describe the dispersion properties of a plasma system with temperature anisotropy. Assuming, for example, the electrons anisotropic, with a bi-Maxwellian velocity distribution function

$$F(v_\parallel, v_\perp) = \frac{1}{\pi^{3/2} v_{\mathrm{th},\parallel} v_{\mathrm{th},\perp}^2} \exp\left[-\left(\frac{v_\parallel^2}{v_{\mathrm{th},\parallel}^2} + \frac{v_\perp^2}{v_{\mathrm{th},\perp}^2}\right)\right], \tag{2.1}$$

the dispersion relation for the transverse EM modes is found to be [3]

$$1 + \frac{\omega_{\mathrm{p}}^2}{\omega^2}\left[A + (A+1)\frac{\omega}{k v_{\mathrm{th},\parallel}} Z\left(\frac{\omega}{k v_{\mathrm{th},\parallel}}\right)\right] = \frac{k^2 c^2}{\omega^2}, \tag{2.2}$$

where $\omega_{\mathrm{p}} = (4\pi n e^2/m)^{1/2}$ is the plasma frequency and $\Omega = |e| B_0/(mc)$ is the absolute value of the electron gyrofrequency. The dispersion equation (2.2) is derived in terms of the Fried and Conte [21] plasma dispersion function $Z(f)$ and, for a finite temperature anisotropy,

$$A = \frac{v_{\mathrm{th},\perp}^2}{v_{\mathrm{th},\parallel}^2} - 1 = \frac{T_\perp}{T_\parallel} - 1 > 0, \tag{2.3}$$

(2.2) admits aperiodic (non-resonant, $\omega_{\mathrm{r}} = 0$) and purely growing solutions ($\omega_{\mathrm{i}} > 0$).

2.2. Counterstreaming plasmas with temperature anisotropy

Here we extend the analysis and consider a counterstreaming plasma system with intrinsic temperature anisotropies as described by the following distribution function:

$$F(v_\parallel, v_\perp) = \frac{1}{2\pi^{3/2} v_{\mathrm{th},\parallel} v_{\mathrm{th},\perp}^2} \exp\left[-\frac{v_\perp^2}{v_{\mathrm{th},\perp}^2}\right] \left[\exp\left[-\frac{(v_\parallel + v_0)^2}{v_{\mathrm{th},\parallel}^2}\right] + \exp\left[-\frac{(v_\parallel - v_0)^2}{v_{\mathrm{th},\parallel}^2}\right]\right]. \tag{2.4}$$

In order to avoid space-charge effects and to decouple the electromagnetic and electrostatic branches [22, 23], the counterstreams are assumed to be symmetric with the same density, bulk velocity and thermal parameters.

The linearized Vlasov–Maxwell equations are used again to find the dispersion relation for the transverse EM mode propagating parallel to the streaming direction ($\mathbf{k} \parallel \mathbf{v}_0$) [14]

$$1 + \frac{\omega_\mathrm{p}^2}{\omega^2}\left\{A + \frac{1}{2}(A+1)[f_1 Z(f_1) + f_2 Z(f_2)]\right\} = \frac{k^2 c^2}{\omega^2}, \tag{2.5}$$

where the arguments of the plasma dispersion function [21] are given by

$$f_{1,2} = \frac{\omega \mp k v_0}{k v_{\mathrm{th},\parallel}}. \tag{2.6}$$

For $v_0 = 0$, the arguments in (2.6) become identical, $f_1 = f_2 = f = \omega/(k v_{\mathrm{th},\parallel})$, and (2.5) simplifies to (2.2). In [14], it was shown that the dispersion relation (2.5) admits unstable solutions of Weibel type and, for a sufficiently large plasma temperature, the growth rates exhibit slope breaks for wave-numbers larger than a threshold value. In the next section, we proceed to a more detailed investigation of the numerical solutions.

3. Numerical solutions

In this section, the dispersion relation (2.5) is solved numerically and the unstable solutions are presented for several cases of interest. First, the growth rates and the frequency dispersion curves are displayed in Fig. 1(a) for a counterstreaming plasma with a parallel thermal velocity of the order of the bulk streaming velocity, $v_{\mathrm{th},\parallel} \simeq v_0 = 10^7$ m/s, and large temperature anisotropies, $v_{\mathrm{th},\perp}/v_{\mathrm{th},\parallel} = 10$, 7.5 and 5. These values could be relevant for energetic jet outflows in gamma-ray burst sources [9, 24] or even for large-scale interpenetration formations of galactic and intergalactic plasmas [10], where the growth rates of this Weibel-like instability have been calculated for the first time in [14]. Here, in addition, we show that, for large wave-numbers, in the interval where the growth rates present slope breaks, the aperiodic feature changes, and the instability becomes oscillatory in time with a finite real frequency, $\omega_\mathrm{r} \neq 0$. Therefore, we call this the resonant regime of the Weibel-like instability (driven by a temperature anisotropy), and this is typically encountered in a counterstreaming plasma with intrinsic temperature anisotropies. For comparison, the growth rates of the Weibel instability in a non-streaming plasma are presented in Fig. 1(b).

In Fig. 2 we show the result of diminishing the anisotropy, which yields growth rates with two peaks of maximum growth. While the first maximum peak represents the saturation in the region where the instability is non-resonant, the second peak is obtained for larger wave-numbers and provides now a second distinct

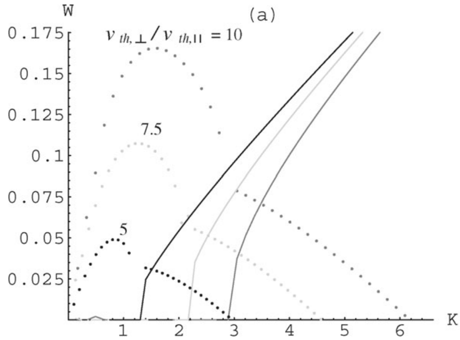

Figure 1. Numerical solutions of (2.5) are shown in panel (a): the real frequency with solid lines and the growth rate with dotted lines, for three large values of the temperature anisotropies $v_{\mathrm{th},\perp}/v_{\mathrm{th},\parallel} = 10, 7.5$ and 5, and a large parallel temperature, $v_{\mathrm{th},\parallel} \simeq v_0 = 10^7$ m/s. For comparison, the aperiodic solutions of (2.2) (non-streaming plasma) are presented in panel (b). The ordinate is scaled as $W = \omega_{\mathrm{r}}/\omega_{\mathrm{p}}$ for frequency (solid lines) and as $W = \omega_{\mathrm{i}}/\omega_{\mathrm{p}}$ for the growth rates (dotted lines in panel (a) and dashed lines in (b)), and the abscissa is scaled as $K = kc/\omega_{\mathrm{p}}$.

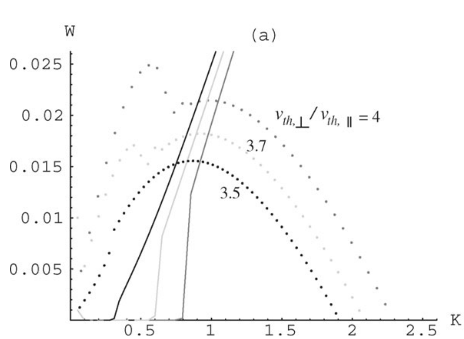

Figure 2. Numerical solutions for (2.5), in panel (a), and for (2.2), in panel (b), for lower anisotropies corresponding to $v_{\mathrm{th},\perp}/v_{\mathrm{th},\parallel} = 4$, 3.7 and 3.5. The remaining parameters are the same as in Fig. 1.

saturation for the resonant instability. The non-resonant Weibel instability saturates due to magnetic trapping of plasma particles when their bounce frequency in the magnetic field of the wave increases to a value comparable to the linear growth rate prior to saturation (isotropization in the magnetic field of the instability) [4]. Fairly speaking, this mechanism could be also efficient for the second saturation of the resonant instability, but this can occur as well due to the linear cyclotron resonance of plasma particles if the Weibel magnetic field is large enough to change the polarization of the wave [25].

For lower anisotropies, as for $v_{\mathrm{th},\perp}/v_{\mathrm{th},\parallel} = 3.5$ in Fig. 2, the first peak decreases significantly, limiting the unstable wave-numbers to very small values. Moreover, when the plasma temperature is sufficiently low, as considered in Fig. 3, the non-resonant wave-number interval is very small and it nearly vanishes, and the instability becomes predominantly resonant. Note that for all cases presented in Figs 1 to 3, the frequency dispersion $\omega_{\mathrm{r}}(k)$ (solid lines) looks almost linear and has a positive slope, and the growth rate is markedly reduced by increasing the

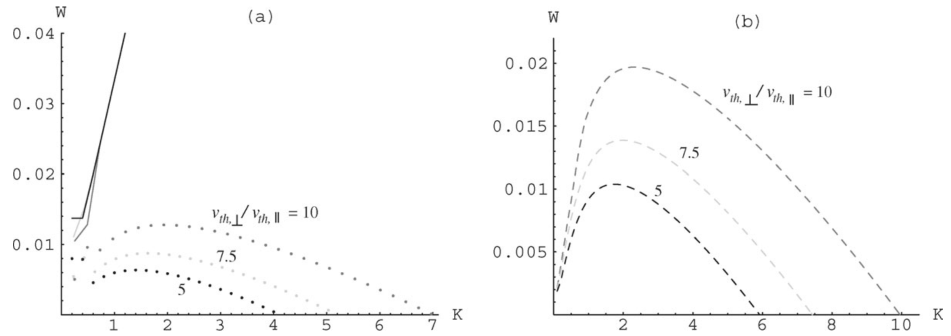

Figure 3. Numerical solutions for (2.5), in panel (a), and for (2.2), in panel (b), for a streaming velocity 10 times larger than the parallel thermal velocity, $v_0 = 10 \times v_{\mathrm{th},\parallel} = 10^7$ m/s. The anisotropy is considered large, $v_{\mathrm{th},\perp}/v_{\mathrm{th},\parallel} = 10$, 7.5 and 5. In this case, the temperature anisotropy instability presented in panel (a) is dominantly resonant, $\omega_{\mathrm{r}} \neq 0$.

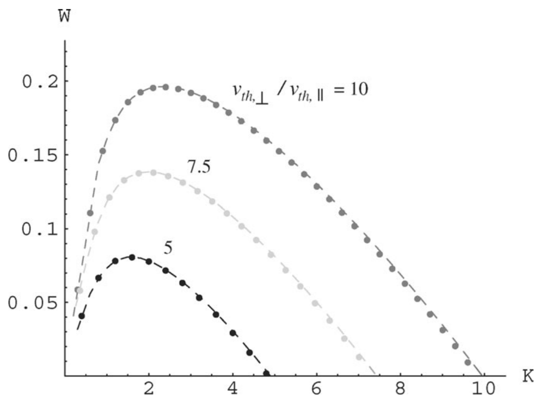

Figure 4. Numerical solutions for (2.5), with dotted lines, and for (2.2), with dashed lines, for a parallel thermal velocity 10 times larger than the streaming velocity, $v_{\mathrm{th},\parallel} = 10 \times v_0 = 10^7$ m/s. The anisotropy is considered large, $v_{\mathrm{th},\perp}/v_{\mathrm{th},\parallel} = 10$, 7.5 and 5. In this case, the temperature anisotropy instability is non-resonant ($\omega_{\mathrm{r}} = 0$), and the growth rates (dotted lines) fit well to those in a non-streaming plasma (dashed lines).

contribution of the streaming velocity with respect to the (transverse) thermal velocity. For instance, the growth rates in Fig. 3 are much smaller than those in Fig. 1, and this is explained by a decrease of the effective anisotropy due to the streaming motion of the plasma, which counterbalances the excess of transverse kinetic energy.

In addition, in Fig. 4 the parallel thermal velocity is considered to be much larger (10 times) than the streaming velocity and the Weibel-like solutions are found to be aperiodic, purely growing, with no real oscillations. This proves the existence of the resonant regime of the Weibel instability only for sufficiently fast streams with a bulk velocity comparable to the parallel thermal velocity.

4. Analytical confirmation

In the previous section, the exact Weibel solutions have been obtained considering large deviations from isotropy, $A > 1$, which widely occur due to a small parallel temperature. As this can be small enough to make the arguments of the plasma dispersion function sufficiently large, $f_{1,2} \gg 1$, here we take the second order in the asymptotic expansion $Z(f \gg 1) \simeq -f^{-1} - (1/2)f^{-3}$, and reduce the dispersion relation (2.5) to

$$\frac{\omega^2}{\omega_\mathrm{p}^2} - 1 - \frac{1}{4}\left[\frac{k^2 v_{\mathrm{th},\perp}^2}{(\omega - kv_0)^2} + \frac{k^2 v_{\mathrm{th},\perp}^2}{(\omega + kv_0)^2}\right] = \frac{k^2 c^2}{\omega_\mathrm{p}^2}. \tag{4.1}$$

The unstable EM modes have subluminal phase velocities, $\omega_\mathrm{r} < kc$, and the growth rates are sufficiently small, $\omega_\mathrm{i} < \omega_\mathrm{p}$, so that $\omega^2 \ll \omega_\mathrm{p}^2 + k^2 c^2$ is neglected in (4.1), which results in a fourth-order dispersion relation, viz.

$$\left(\frac{k^2 c^2}{\omega_\mathrm{p}^2} + 1\right)(\omega^2 - k^2 v_0^2)^2 + \frac{1}{2} k^2 v_{\mathrm{th},\perp}^2 (\omega^2 + k^2 v_0^2) = 0. \tag{4.2}$$

For arbitrary values of the transverse temperature and the streaming velocity, (4.2) has complex solutions including unstable modes with a finite oscillation frequency, $\omega_\mathrm{r} \neq 0$. However, the dispersion relation (4.2) is quadratic in ω^2 and we find that only for a transverse temperature sufficiently large, so that

$$v_{\mathrm{th},\perp} > 4v_0, \tag{4.3}$$

and wave-numbers smaller than a threshold value $k < k_\mathrm{t}$ given by

$$k_\mathrm{t} = \frac{\omega_\mathrm{p}}{c}\left[\frac{v_{\mathrm{th},\perp}^2}{16 v_0^2} - 1\right]^{1/2}, \tag{4.4}$$

the equation (4.2) admits an aperiodic purely growing solution of the form

$$\omega = ikv_{\mathrm{th},\perp}\left[\frac{\omega_\mathrm{p}^2}{2(k^2 c^2 + \omega_\mathrm{p}^2)} - \frac{3v_0^2}{v_{\mathrm{th},\perp}^2}\right]^{1/2}. \tag{4.5}$$

Theoretically, the aperiodic solution exists provided that the wave-number is less than a *virtual* cutoff wave-number

$$k_\mathrm{c}^\mathrm{v} = \frac{\omega_\mathrm{p}}{c}\left[\frac{v_{\mathrm{th},\perp}^2}{6 v_0^2} - 1\right]^{1/2}, \tag{4.6}$$

to which the growing solution (4.5) vanishes, $\omega_\mathrm{i}(k_\mathrm{c}^\mathrm{v}) = 0$. In perfect agreement with what we have shown in Figs 1 and 2, the threshold (4.4) is smaller than this cutoff wave-number in (4.6), which is called virtual because (4.5) stops being a solution of the dispersion relation (4.2) exactly at the threshold wave-number. For wave-numbers larger than this threshold, $k > k_\mathrm{t}$, the growth rate line breaks and changes its slope and the instability becomes resonant with $\omega_\mathrm{r} \neq 0$. As a matter of fact, the resonant instability will stabilize for a cutoff wave-number greater than (4.6), and this is confirmed in Figs 1 to 3.

We have no intention to provide here an approximative analytical form for the resonant solution or the cutoff wave-number because we have shown in the previous section that they can be calculated exactly numerically. Most important is that the resonant regime of the Weibel-like mode is confirmed analytically, being associated with wave-numbers larger than a threshold value given in (4.4). When the plasma flows faster, this threshold decreases and the instability becomes dominantly resonant. Such cases are presented in Fig. 3.

On the other hand, the aperiodic non-resonant solutions exist only for a large transverse temperature given by the condition (4.3). Despite the fact that the accuracy of the condition (4.3) is limited by the second-order approximation used for the plasma dispersion function, this condition approximates quite well what we have found exactly numerically in the previous section; see for example Fig. 2, where the aperiodic purely growing solutions exist only for $v_{\mathrm{th},\perp} \gtrsim 3.5v_0$. Moreover, if the streams are slow, with a bulk velocity much less than the parallel thermal velocity, see for instance Fig. 4, the instability is purely growing and non-resonant.

5. Conclusions

In this work we have investigated the stability properties of a counterstreaming plasma system with intrinsic thermal anisotropies, namely the Weibel-like instability driven along the streams by an excess of transverse temperature. Because of the streaming motion of the plasma, the Weibel unstable mode behaves differently from the classical Weibel instability in a non-streaming plasma. Thus, for streaming velocities sufficiently large, and for wave-numbers larger than a threshold value that depends on the temperature anisotropy, the growth rates exhibit slope breaks. We have shown that exactly in this wave-number interval, the purely growing instability changes and becomes a fully propagating mode that propagates not only in space but also in time, $\omega_{\mathrm{r}} \neq 0$. This can be called the *resonant* regime of a Weibel-like unstable mode, and it is typically encountered in a counterstreaming plasma system with intrinsic temperature anisotropies.

Acknowledgements

ML acknowledges financial support from the Research Foundation Flanders – FWO Belgium. MED thanks Vetenskapsradet and the DFG (through the Forschergruppe 1048) for financial support. These results were obtained in the framework of the projects GOA/2009-009 (K. U. Leuven), G.0304.07 (FWO-Vlaanderen) and C 90205 (ESA Prodex 9). Financial support by the European Commission through the SOLAIRE Network (MTRN-CT-2006-035484) is gratefully acknowledged. The numerical results were obtained on the HPC cluster VIC of the K. U. Leuven.

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